Optimizing modular product design for reconfigurable manufacturing

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The problem of optimizing modular products in a reconfigurable manufacturing system is addressed. The problem is first posed as a generalized subset selection problem where the best subsets of module instances of unknown sizes are determined by minimizing an objective function that represents a trade-off between “the quality loss due to modularization” and the cost of reconfiguration while satisfying the problem constraints. The problem is then formulated and solved as an integer nonlinear programming problem with binary variables. The proposed method is applied to the production of a modular drive system composed of a DC motor and a ball screw. The study is a first attempt toward developing a systematic methodology for manufacturing modular products in a reconfigurable manufacturing system.

Keywords: Modular products, product design, reconfigurable manufacturing, optimization, integer programming

1. Introduction

A new manufacturing paradigm called reconfigurable manufacturing systems (RMS) is emerging to address the needs caused by rapidly changing markets and rapid introduction of new products (Koren et al., 1999). A reconfigurable manufacturing system is designed for rapid adjustment of production capacity and functionality, in response to new circumstances, by rearrangement or change of its components. These new systems provide exactly the functionality that is needed exactly when it is needed (Mehrabi et al., 2000). Therefore, a RMS is designed to be easily reconfigured such that it is able to process a family of parts and accommodate new and unanticipated changes in the product design and processing needs.

The utility of a RMS is greatly increased if it is designed for production of modular products, where the combinations of individual modules form the product. The term modularity is used to describe the use of common units to create product variants (Huang and Kusiak, 1998). Through modularity, the number of parts to be manufactured for a product family may be significantly reduced while achieving sufficient variety by combination of different modules (see Fig. 1). In general each module may have more than one instance. The different instances provide the sizes and capabilities that are required by the desired product variety, and together they form the part family. The modular products in the part family are all the variants (i.e., $A_i + B_j; i = 1, 2, 3; j = 1, 2$) shown in Fig. 1. A particular configuration of the RMS for a particular module can then be used to produce a particular instance of the part family (see Fig. 2). The first production line (RMS-A) can be quickly and cost effectively reconfigured, as needed in response to market demand to produce any instance of module A (i.e., $A_i; i = 1, 2, 3$). Similarly, the second production
line (RMS-B) can be reconfigured to produce either B1 or B2. This enables the manufacturer to be responsive to changing and unpredictable demand. It also requires that the product be designed in a modular manner (i.e., as the combination of two modules $A_i$ and $B_j$ in this example).

The Nippondenso panel meter design (Aoki, 1980) is cited in Kusiak (1999) as a powerful example that clearly illustrates the benefits of modularity. The old panel meter was redesigned with six standard modules. Through redesign the number of parts was significantly reduced; e.g., the number of voltage regulators was reduced from 20 to three, the number of bimetals was reduced from eight to four and so on. The combination of six modules resulted in 288 different models, of which 40 were produced. (With the previously considered number of alternatives, the number of possible models were 23040.) As this example illustrates, the benefits of modularity include: economies of scale; increased feasibility of product/component change; increased product variety; reduced lead time; easier product diagnosis, decoupled risks, maintenance, repair and disposal.

Despite these clear benefits, a formal theoretical approach to modularity is still lacking (Kusiak, 1999), and designers are often skeptical regarding the advantages of modularity. This is largely due to the inferior performance obtained by modular designs compared to their custom built optimal alternatives (Cakmakci and Ulsoy, 2000; Ulrich and Seering, 1989). Recently, there have been some attempts to address various issues in modular product design such as planning for commonality, optimizing the degree of commonality and finding the optimum settings for the common modules (Fujita et al., 1999; Gonzales-Zugasti and Otto, 2000; Martin and Ishii, 1997). Fujita et al. (1999) proposed an optimization approach to designing modular products from existing modules using an integer-programming formulation. Gonzales-Zugasti and Otto (2000) presented a general method for designing families of products built onto modular platforms. These modular platforms allow for the use of both existing and new modules. Optimizing modular products in a RMS has not been addressed before. As mentioned above, in RMS each module instance required for a particular product variant is produced by a different configuration. Therefore, the design of modular products should consider the cost of reconfiguration, in addition to other issues related to modularity.

This paper addresses the problem of manufacturing modular products in a RMS environment. For a modular product that is to be manufactured in a RMS, the performance of a custom built alternative can be approached if the number of module instances (i.e., different sizes or capabilities) is increased indefinitely. However, this is neither practical, nor economical since each instant requires a different configuration. Therefore, a major issue in designing and manufacturing modular products in a RMS is to determine optimum number of module instances and the selection of the optimum subset of module instances from a large (possibly infinite) number of

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**Fig. 1.** A typical modular product with two types of modules.

**Fig. 2.** Manufacturing a modular product on a reconfigurable manufacturing system.
alternatives. In the current work this problem is posed first as a subset-selection problem where the cost of selecting a subset from the set of all possible alternatives is to be minimized. The problem is then transformed into a nonlinear programming problem that can be solved efficiently to aid in optimum planning for modular production in a RMS environment. This study is a first attempt in designing modular products manufactured in a RMS environment. The proposed mathematical formulation is also novel in the sense that it facilitates an efficient solution through the use of binary variables.

2. General problem formulation

The basic design problem considered can be described as a constrained optimization problem where the design variables can be divided into a number of groups which represent modules that then make up the complete product. For example, a powertrain is considered to be composed of the engine and transmission, and the design variables can be grouped accordingly. Let us assume, without loss of generality, that the set of design variables is decomposed into two groups represented by the vectors \( \mathbf{d}_1 \) and \( \mathbf{d}_2 \). The parameter set \( \mathbf{p} \) represents the performance requirements specified by the customer. The custom-made product is obtained as the solution to the following optimization problem:

\[
\min_{\mathbf{d}_1, \mathbf{d}_2} W(\mathbf{d}_1, \mathbf{d}_2, \mathbf{p})
\]  

subject to constraints

\[
g(\mathbf{d}_1, \mathbf{d}_2, \mathbf{p}) \leq 0
\]  

and side conditions (bounds)

\[
d_1^1 \leq d_1 \leq d_1^u \quad \text{and} \quad d_2^1 \leq d_2 \leq d_2^u
\]  

Vectors of design variables and the parameter vector \( \mathbf{p} \) belong to the following finite or infinite vector spaces:

\[
\mathbf{d}_1 \in \mathbf{S}_1, \quad \mathbf{d}_2 \in \mathbf{Q}_2, \quad \mathbf{p} \in \mathbf{P}
\]  

The design task is to determine the optimum values of design variables for a given parameter set, and a given objective function \( W \). This formulation assumes that there is a single objective function, or a combined one in the case of multiple objectives. In general, the parameters may vary due to changing customer requirements or preferences. In the case of a custom-made product, the manufacturing system should be set up to produce a different product for each parameter set representing a different specification. For example, based on customer preferences we might custom design a drive system by designing both a motor and transmission to meet those requirements. In a RMS environment, this may lead to a large number of configurations, which may not be practical or economical to utilize the benefits of RMS.

If the product to be manufactured is modular in nature, then, the number of configurations for each module may significantly be reduced since modularity provides desired variety of the product through different combinations of modules. The modular product is obtained by solving the following discrete optimization problem:

\[
\min_{\mathbf{d}_1, \mathbf{d}_2} W(\mathbf{d}_1, \mathbf{d}_2, \mathbf{p})
\]  

subject to constraints

\[
g(\mathbf{d}_1, \mathbf{d}_2, \mathbf{p}) \leq 0
\]  

and side conditions (bounds)

\[
d_1^1 \leq d_1 \leq d_1^u \quad \text{and} \quad d_2^1 \leq d_2 \leq d_2^u
\]  

In the modular product case, the design variables are to be selected from among a finite number of discrete sets

\[
\mathbf{d}_1 \in \mathbf{S}_i, \quad i = 1, 2, \ldots, N_S, \\
\mathbf{d}_2 \in \mathbf{Q}_j, \quad j = 1, 2, \ldots, N_Q
\]  

where \( N_S \) and \( N_Q \) are the total numbers of all possible subsets, \( \mathbf{S}_i \) and \( \mathbf{Q}_j \), respectively, which can be formed from the sets representing the discrete design domain for \( \mathbf{d}_1 \) and \( \mathbf{d}_2 \), respectively. Thus,

\[
\mathbf{S}_i \subseteq \mathbf{S} \quad \mathbf{Q}_j \subseteq \mathbf{Q}
\]  

The percent quality loss due to modularization for a particular parameter set can be defined as

\[
\beta(\mathbf{p}, \mathbf{S}_i, \mathbf{Q}_j) = 100 \left( \frac{W^+(\mathbf{p}, \mathbf{S}_i, \mathbf{Q}_j) - W^*(\mathbf{p})}{W^*(\mathbf{p})} \right)
\]  

where \( W^+ \) and \( W^* \) are the values of the objective functions at the solutions to optimization problems described by Equations (5)–(7), and Equations (1)–(3), respectively. Clearly, different subsets will result in different quality loss for a given parameter set. Therefore, the subsets resulting in a minimum quality
loss have to be selected. In general as the size of the subsets increases the quality loss decreases. However, this means increasing number of configurations for the RMS, which has to be accounted for in optimizing the overall cost.

The total number of possible subsets for a two-module problem is given as

\[ n_T = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} C_i^N C_j^N \]  

(11)

where \( N_1 \) and \( N_2 \) are the sizes of sets \( S \) and \( Q \), respectively, \( n_1 \) and \( n_2 \) are the maximum allowable sizes of \( S_i \) and \( Q_j \), and \( C_i^N \) denote combinations of \( J \) objects taken \( i \) at a time.

A subset-selection problem is usually formulated to solve challenging routing and scheduling problems as well as a class of regression problems. A subset selection method was used to determine the optimal number and location of actuators for controlling structural vibrations (Ruckman and Fuller, 1995). Various subset selection algorithms have also been proposed to determine the optimal number of classes and their intervals for selective assembly (Kwon et al., 1999). A general approach to solve these types of problems starts by determining a number of interesting routes or subsets of customers (in case of a routing problem), subsets of tasks (in the case of a scheduling problem) or subsets of parameters (in the case of regression analysis); then, selects, among these subsets, a collection that allows optimization of a given objective while satisfying the problem constraints (Boctor and Renaud, 2000). In the classical subset-selection problem, the size of the subsets is usually known. Efficient algorithms which consist of an exhaustive search constrained by bounding rules and guided by a search-ordering procedure have been developed to solve such problems (Boyce et al., 1974).

The problem considered here differs from the classical subset selection problem due to the following reasons: (1) The sizes of the subsets (i.e., the number of module instances) are unknown, though generally, an upper limit is imposed on the size of the subsets. (2) The subset to be selected is a combination of subsets of each module. Thus, the problem at hand can be considered as a multidimensional (dimension being equal to the number of modules) generalization of classical subset selection. In the current work, this subset selection problem is formulated as a nonlinear programming problem as described in the next section.

3. Formulation as a nonlinear programming problem

The problem of selecting the best subsets for each module can be formulated as an integer nonlinear programming problem by using binary variables, \( x_i^k \), and \( y_j^k \), which are defined as follows:

\[ x_i^k = \begin{cases} 
1 & \text{if the } i \text{th instant of module 1 is selected} \\
0 & \text{otherwise}
\end{cases} \]

\[ y_j^k = \begin{cases} 
1 & \text{if the } j \text{th instant of module 2 is selected} \\
0 & \text{otherwise}
\end{cases} \]

Let the quality loss associated with the modular product obtained by selecting the \( i \)th instant of module 1 and \( j \)th instant of module 2 for the \( k \)th parameter, be denoted as \( \Phi_{ij}^k \), which can be determined as:

\[ \Phi_{ij}^k = W_{ij}^k - W_{ij}^{*k} \]

where \( W_{ij}^k \) is the value of the objective function in Equation (5) for the \( i \)th instant of module 1 and \( j \)th instant of module 2 for the \( k \)th parameter set, and \( W_{ij}^{*k} \) is the value of the objective function in Equation (1) for the optimum design.

Let \( n_1 \) and \( n_2 \) denote the numbers of selected instances for each module, respectively (the sizes of subsets). Then, the problem to be solved can be written as

\[ \min \frac{1}{P} \sum_{k=1}^{P} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \phi_{ij}^k x_i^k y_j^k + C_1 n_1 + C_2 n_2 \]

(12)

subject to

\[ g_{ij}^k x_i^k y_j^k \leq 0 \quad i = 1, 2, \ldots, N_1 \]

\[ j = 1, 2, \ldots, N_2 \]

\[ k = 1, 2, \ldots, P \]

(13)

\[ n_1 \leq \bar{n}_1 \]

(14)

\[ n_2 \leq \bar{n}_2 \]

(15)

\[ \sum_{i=1}^{N_1} x_i^k = 1 \quad k = 1, 2, \ldots, P \]

(16)
\[ \sum_{j=1}^{N_i} y_j^k = 1 \quad k = 1, 2, \ldots, P \]  

(17)

where \( P \) is the number of different parameter sets considered. Equations (16) and (17) guarantee that exactly one module is selected for each parameter set. The constants C1 and C2 are the relative costs of reconfiguration for modules 1 and 2, respectively. The objective function in Equation (12), represents a trade-off between the average quality loss and the cost of reconfiguration. In order to complete the formulation, the subset sizes \( n_1 \) and \( n_2 \) must be expressed in terms of the decision variables \( x_i^j \) and \( y_j^k \). This is accomplished by defining the following auxiliary integer variables

\[ s_1^i = \sum_{k=1}^{P} x_k^i \quad i = 1, 2, \ldots, N_1 \]  

(18)

\[ s_2^j = \sum_{i=1}^{N_1} y_i^j \quad j = 1, 2, \ldots, N_2 \]  

(19)

\[ n_1 = 1 \quad \text{if} \quad s_1^i > 0 \quad i = 1, 2, \ldots, N_1 \]  

(20)

\[ n_2 = 1 \quad \text{if} \quad s_2^j > 0 \quad j = 1, 2, \ldots, N_2 \]  

(21)

The auxiliary variables \( s_1^i \) and \( s_2^j \) are counters which track how many times each module is selected. If a particular instant of module 1 (model 2) is selected at least for one parameter set, then it is strictly positive, and hence, \( n_1 = 1 \) \( (n_2 = 1) \). If a particular module is not selected for any parameter set, then, it is zero. Thus, the subset sizes can be obtained as

\[ n_1 = \sum_{i=1}^{N_1} n_1^i \]  

(22)

\[ n_2 = \sum_{j=1}^{N_2} n_2^j \]  

(23)

In order to transform Equations (20) and (21) which contain if statements into standard constraint form, they can be equivalently written by the following equations (Winston, 1994):

\[ 1 - n_1^i \leq K_m z_i^l \quad i = 1, 2, \ldots, N_1 \]  

(24)

\[ s_1^i \leq K_m (1 - z_i^l) \]  

(25)

where \( z_i^l \) and \( z_j^m \) are binary variables, and \( K_m \) is a large number. For the problem considered \( K_m \geq P \). Since binary integer variables are used as decision variables, the solution method is quite efficient in identifying the best subsets for the given objective function. The formulation is quite general to include such constraints as pre-selecting a particular module. For example, if the manufacturer wants to include \( i \)-th instant of module 1, then, an additional constraint is specified as \( x_i^j = 1, k = 1, 2, \ldots, P \).

3.1. Special case: fixed subset size

Let the size of the subsets \( S_i \) and \( Q_j \) be fixed at \( I \) and \( J \), respectively. This situation arises when the manufacturer has a preference for the number of module instances (equivalently number of different configurations for the RMS). In this case, the problem reduces to that of finding the minimum cost product for each parameter set. The complexity of the problem, as measured by the number of possible subsets to be searched is given as

\[ n_T = C_{I}^{N_1} C_{J}^{N_2} \]  

(26)

3.2. Special case: fixed subset

In case the discrete design domain is small, the whole sets \( S \) and \( Q \) can be selected as the subsets, and the problem is reduced to custom-made design described by Equations (1)–(4).

4. Illustrative example: DC motor and transmission system

In order to illustrate the proposed methodology a simple modular assembly problem is studied. The assembly considered is a drive system, which consists of a DC Motor, a gearbox and a ball screw. The objective is to assemble a system with minimum power requirement (smallest size) while satisfying the constraint that it is able to provide a maximum acceleration for a given load mass subject to a given load force. The maximum power required by the motor-ball screw assembly is given by Fussel and Taft (1995).
where $T_p$ is the peak torque at the motor shaft, and $V_m$ is the maximum velocity of the load. The peak torque is given by

$$T_p = \left( J_m + m_l \left( \frac{P_s}{2\pi n_k} \right)^2 + J_f \right) n_k \frac{\ddot{\theta}_{sm}}{n_k^2} + F_{fr} \left( \frac{P_s}{2\pi n_k} \right) + B_L \left( \frac{P_s}{2\pi n_k} \right) V_m + F_L \left( \frac{P_s}{2\pi n_k} \right)$$

(28)

subject to

$$\frac{T_p}{T_{\text{max}}} - 1 \leq 0$$

(30)

and Equations (27) and (28), where $T_{\text{max}}$ is the allowable peak torque for the motor.

In the modular design, the drive system is assembled from a subset of available DC motors and ball screws. The problem is to determine the best subsets, which result in best possible matches for a given load, $F_L$. As the subset sizes increase the performance of the modular design will approach that of the custom design. However, the cost of reconfiguration, and production will also increase. The objective function given by Equation (12) helps in finding the optimum subsets. Here, $C_1$ and $C_2$ can be considered as the relative costs of reconfiguration for the motor and ball screw units, respectively. Note that the capacity issue is not considered in the current problem. Furthermore, it is assumed that all values of parameter sets are assumed to be equally likely. If, however, probability distributions of parameter sets (i.e., customer preferences, or demand) are known, a stochastic optimization problem can be formulated.

### 4.1. Solution by nonlinear programming

To illustrate the methodology, the proposed nonlinear integer programming formulation is used to solve the following modular assembly problem. Let’s assume that a drive system has to be assembled from available motors and ball screws. For simplicity and without loss of generality, a limited number of motor and ball screw units manufactured by leading manufacturers are considered to be available for selection. The objective of the modular assembly is to be able to assemble near optimal drive systems for the load values considered with a smaller number of motor and ball screw units. It is assumed that a particular series, which have high torque-to-inertia ratios, will be used. There are four available motors with inertia values of 0.071, 0.093, 0.11, and 0.13 kgm$^2$ × 10$^{-3}$ (labeled as M1, M2, M3, and M4, respectively). It is also assumed that cylindrical nut, ball screws with a ball circle diameter of 25 mm are used. There are three ball screws with pitch sizes of 5, 10, and 25 mm (labeled as T1, T2, T3, respectively). Thus, $N_1 = 4$, and $N_2 = 3$. The limits of subset sizes are the same as the set sizes, i.e., $h_1 = 4$, and $h_2 = 3$. The load force is
Table 1. Nonlinear programming results for various values of relative configuration costs

<table>
<thead>
<tr>
<th>Selected motors</th>
<th>C1 = 0.01, C2 = 0.001</th>
<th>C1 = 0.01, C2 = 10</th>
<th>C1 = 0.01, C2 = 0.01</th>
<th>C1 = 0.1, C2 = 0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2, M3, M4</td>
<td>M1, M2, M3</td>
<td>M3, M4</td>
<td>M4</td>
<td></td>
</tr>
<tr>
<td>Selected ball screws</td>
<td>T1, T2</td>
<td>T1</td>
<td>T1, T2</td>
<td>T1, T2</td>
</tr>
<tr>
<td>Average percent quality loss</td>
<td>7.4</td>
<td>22.6</td>
<td>7.5</td>
<td>8.1</td>
</tr>
</tbody>
</table>

due to modularization (%)

assumed to take on values of 1000, 2000, 3000, 4000, and 5000 N. Therefore, \( P = 5 \). The rest of the data is \( m_c = 100 \) kg, \( V_m = 0.254 \) m/s, \( \beta_{\text{nom}} = 3000 \) rad/s² \( F_g = 500 \) N, \( B_L = 10 \) Ns/m, \( L_s = 2 \) m, \( \rho = 7800 \) kg/m³ and the screw inertia is given by \( J_s = \rho n D_s^4 L_s/32 \) where \( \rho \) is the density of ball screw material.

The solutions obtained by using various values of relative configuration costs are given in Table 1. The average percent quality loss (APQL) is defined as

\[
\tilde{\beta} = \frac{1}{P} \sum_{k=1}^{P} \beta^k
\]

where \( \beta^k \) is the quality loss incurred by using the subsets obtained from the solution to the modular production problem for the \( k \)th parameter set, and can be obtained from Equation (10).

If it is assumed that the DC motor and ball screw units are to be manufactured in a RMS, a reference to Fig. 2 can be made again to depict a particular manufacturing strategy deduced from the nonlinear programming solution. In this example, motor and ball screw units are the modules of type A and B, respectively. If the first set of weights is adopted, for example, then, Motor units will be produced on the first production line (RMS-A) with Configurations 1, 2, and 3, designed to produce M2, M3, and M4, respectively. Similarly, Ball screw units will be produced on the second production line (RMS-B) with Configurations 1 and 2, designed to produce T1, and T2, respectively.

As expected, as the relative configuration cost for a module is increased, the number of required configurations for that module decreases. Clearly, this is achieved at the expense of incurring some quality loss due to modularization. However, by properly adjusting the optimization parameters (i.e., relative configuration weights) this loss can be kept at a desirable level while reducing the overall cost of manufacturing.

5. Summary and conclusions

The problem of optimum selection or design of module instances for a modular product, manufactured in a RMS environment, has been addressed. The problem is first posed as a generalized subset selection problem where the best subsets of unknown sizes are to be found which minimizes an objective function while satisfying the problem constraints. The problem is then formulated and solved as an integer nonlinear programming problem. The proposed formulation is based on finding a trade-off between the quality loss due to modularity and the cost of reconfiguration. The method was applied to a modular assembly problem of a drive system composed of a DC motor and a ball screw, and found to be very efficient in determining optimum subsets of each module from a given set.

With the methodology, an adequate trade-off between the product quality and manufacturing efficiency can be made since the quality loss due to modularization can be controlled by adjusting the optimization parameters. Thus, the proposed method can be used as a systematic tool in selection of module instances. The formulation can easily be modified according to the needs. For example, it is straightforward to include other cost elements such as economies of scale.

Integer nonlinear programming methods are in general computationally expensive. Though the computational efficiency is greatly enhanced by the use of binary variables, for a large number of modules, or parameter sets, the problem becomes very complex and may require extensive computational resources. In this case, some heuristics may be useful to speed up the solution.

The formulation can be generalized for products
made up of more than two modules. Though the complexity of the problem will increase significantly, this extension is straightforward. The proposed methodology can also be used in designing modular products (i.e., breaking-up a product into different modules). Once various alternative designs are generated, examining the average percent quality loss due to modularization can distinguish competing modular designs for a given modular product. In the current work, it is assumed that there is equal demand for each parameter set (i.e., customer preference or requirement). It is straightforward to account for unequal, but deterministic demand by including different weights for each parameter set in the objective function. It will be interesting to reformulate the problem as a stochastic optimization problem, which accounts for random customer requirements.

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References


